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TRAJECTORY DETERMINATION BY A LEAST-SQUARES DIFFERENTIAL CORRECTION OF THREE-DEGREE-OF-FREEDOM ACCELERATIONS

By

Paul O. Hurst

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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COMPUTATION DIVISION

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

A	Magnitude of the component of the aerodynamic force along the vehicle's axis.
A_L	Component of acceleration in the vehicle's longitudinal axis due to thrust and aerodynamic forces.
A_{ei}	Area of exhaust $i = 1, 2, 3, 4, 5, 6, 7, 8$
A_{ti}	Area of throat $i = 1, 2, 3, 4, 5, 6, 7, 8$
a	Semi-major axis of the reference ellipsoid
b	Semi-minor axis of the reference ellipsoid
C10 C11 C12 C20 C21 C30 C31 C32 B10 B20 B21	Constants used in the expansion of the gravitational potential of the earth in the ρ, ψ coordinate system.
A_o	Coefficient of $(X-\bar{X})$ in β_ξ
B_o	Alpha control coefficient in pitch
$\left.\begin{matrix} C_o \\ C_1 \end{matrix}\right\}$	Slant altitude guidance coefficients
$C_1^{(1)}, C_2^{(1)}, C_3^{(1)}$	Direction cosines in the direction of the vehicle's axis.
CC	Location of the swivel point in meters from base.
C_{Do}	Coefficient of drag
C_{Fi}	Coefficient of thrust
CG	Center of gravity

LIST OF SYMBOLS (Cont'd)

C_n^1	Coefficient of normal force
CP	Center of pressure
D	Diameter of the vehicle
$\left.\begin{matrix}E_o\\E_1\end{matrix}\right\}$	Cross range guidance coefficients
$F(0)_i$	Thrust of outboard engines $i = 1, 2, 3, 4$
$F(I)_i$	Thrust of inboard engines $i = 5, 6, 7, 8$
$E_e(0)_i$	Exhaust thrust of outboard engines $i = 1, 2, 3, 4$
$F_e(I)_i$	Exhaust thrust of inboard engines $i = 5, 6, 7, 8$
$F_1^{(1)}, F_2^{(1)}, F_3^{(1)}$	Direction cosines of the outboard thrust vector.
G_ρ	Component of gravity in direction of radius vector.
G_ψ	Component of gravity normal to radius vector.
H	Distance from the earth's surface to the vehicle (altitude).
H_o	Altitude of the launch site.
I_1^*	Presetting of ξ_M in meters/second
I_2^*	Presetting of $\dot{\xi}_M$ in meters
J_1^*	Presetting of η_M in meters/second
J_2^*	Presetting of $\dot{\eta}_M$ in meters
K	Firing azimuth positive counterclockwise from west.
K_1	Restoring moment coefficient due to angle of attack.

* Note these are always Zero

LIST OF SYMBOLS (Cont'd)

$(K_2)_P$	Restoring moment coefficient in pitch due to thrust.
$(K_2)_Y$	Restoring moment coefficient in yaw due to thrust.
M	Instantaneous mass in kg Sec^2/meter.
MOI	Moment of inertia in pitch and yaw.
MACH	Mach Number
N'	Slope of the normal force.
$N_1^{(1)}, N_2^{(1)}, N_3^{(1)}$	Direction cosines of the normal force vector in the X", Y", Z" system.
P	Atmospheric pressure
P_{ci}	Chamber pressure $i = 1, 2, 3, 4, 5, 6, 7, 8$
PR	Number of perturbed trajectories
Q	Dynamic pressure
R	Radius vector from the center of the earth to the CG of the vehicle.
R_I	Distance along the radius vector from the earth's center to the earth's surface.
R_o	Magnitude of radius vector from the center of the earth to the launch site.
$V_1^{(1)}, V_2^{(1)}, V_3^{(1)}$	Direction cosines of the relative velocity vector
V_1, V_2, V_3	Components of the vehicle's relative velocity in the X_p, Y_p, Z_p coordinate system
V_R	Vehicle's relative velocity
V_S	Velocity of sound
W_1, W_2, W_3	Components of the wind in the X_p, Y_p, Z_p coordinate system

LIST OF SYMBOLS (Cont'd)

X_p, Y_p, Z_p	Space-Direction-Fixed, Right-Handed coordinate system with origin at the initial position of the launch site.
X'', Y'', Z''	Space-Direction-Fixed, Right-Handed coordinate system with origin at the center of the earth.
X_o, Y_o, Z_o	Initial coordinates of the launch site in the X'', Y'', Z'' coordinate system.
X, Y, Z	Coordinates of vehicle in X_p, Y_p, Z_p coordinate system.
X_m, Y_m, Z_m	Magnitude of the measured components of displacement in the X_p, Y_p, Z_p system.
X_g, Y_g, Z_g	Magnitude of the gravitational components of displacement in the X_p, Y_p, Z_p system.
$\dot{X}_o$, $\dot{Y}_o$, $\dot{Z}_o$	Components of the initial velocity of the vehicle in the X'', Y'', Z'' system.
$\dot{X}$, $\dot{Y}$, $\dot{Z}$	Components of the vehicle's velocity in the X_p, Y_p, Z_p coordinate system.
$\dot{X}_m$, $\dot{Y}_m$, $\dot{Z}_m$	Measured components of velocity in the X_p, Y_p, Z_p system.
$\dot{X}_g$, $\dot{Y}_g$, $\dot{Z}_g$	Gravitational components of velocity in the X_p, Y_p, Z_p system.
$\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$	Components of the vehicles acceleration in the X_p, Y_p, Z_p coordinate system.
$\ddot{X}_m$, $\ddot{Y}_m$, $\ddot{Z}_m$	Components of the vehicle's acceleration in the X_p, Y_p, Z_p system due to measured accelerations.
$\ddot{X}_g$, $\ddot{Y}_g$, $\ddot{Z}_g$	Components of the vehicles acceleration in the X_p, Y_p, Z_p system due to gravitation.
α	Angle of attack
α_ξ	Angle of attack in side plane
α_ζ	Angle of attack in roof plane

LIST OF SYMBOLS (Cont'd)

β_o	Difference between Geodetic and Geocentric latitude of launch site.
β_ξ	Rudder angle in body (pitch plane)
β_ζ	Rudder angle in yaw plane
$\left.\begin{matrix}\delta_1\\ \delta_2\end{matrix}\right\}$	The rotation of the ξ, η plane due to the deflection of the vertical at the launch site.
ϵ	Angle between the axis and the horizon at the launch site.
$\bar{\eta}$	Vehicle's measured displacement along the η axis.
$\bar{\dot{\eta}}$	Vehicle's measured velocity along the η axis.
ν	Angle in the roof plane between the projection of the velocity vector into the roof plane and the X_p, Y_p plane.
ξ, η, ζ	Coordinates of the vehicle in the ξ, η, ζ coordinate system.
ξ_m, η_m, ζ_m	Magnitudes of the measured components of displacement along the ξ, η, ζ axis.
ξ_g, η_g, ζ_g	Magnitudes of the gravitational components of displacement along the ξ, η, ζ axis.
$\dot{\xi}_o, \dot{\eta}_o, \dot{\zeta}_o$	Components of the initial velocity of the vehicle in the ξ, η, ζ system.
$\dot{\xi}, \dot{\eta}, \dot{\zeta}$	Components of the vehicle's velocity in the ξ, η, ζ coordinate system.
$\dot{\xi}_m, \dot{\eta}_m, \dot{\zeta}_m$	Measured components of velocity in the ξ, η, ζ system.
$\dot{\xi}_g, \dot{\eta}_g, \dot{\zeta}_g$,	Gravitational components of velocity in the ξ, η, ζ system.
ρ	Density
τ	Roof plane

LIST OF SYMBOLS (Cont'd)

Φ_o	Geodetic latitude of launch site
Φ	Instantaneous Geodetic latitude
χ	Angle between the roof plane and the X_p, Y_p plane
$\bar{\chi}$	Gyro-tilt program
ψ_o	Geocentric latitude of launch site
ψ	Instantaneous Geocentric latitude
Ω_1, Ω_2, Ω_3	Components of the angular velocity
$\Omega_1^{(1)}$, $\Omega_2^{(1)}$, $\Omega_3^{(1)}$	Components of the unit vector of the angular velocity.
ω	Angular velocity of the earth.

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SUMMARY

A method of determining continuous trajectory data is presented. A standard three-degree-of-freedom trajectory, with the differential equations of motion numerically integrated utilizing a fourth-order Runge-Kutta method, is used for the basic trajectory representation. Partial derivatives for the least-squares adjustment are obtained by numerical perturbations of the parameters which generate and control the trajectory. These parameters are then adjusted using large amounts of various types of observed data.

SECTION I. INTRODUCTION

This problem is to obtain a continuous trajectory and associated trajectory parameters from a model that is as close to the actual dynamic case as is possible under the assumptions made. All types of observations that are made consistent with the basic assumptions of the chosen model are to be used in a least-squares differential correction to the parameters that generate or control the trajectory. The set of differential equations describing the motion is the model used since all types of trajectory data, at least in theory, can be used as observations. The differential equations of motion in three degrees of freedom are numerically integrated using a standard fourth-order Runge-Kutta method. This trajectory computation treats the control system as if the thrust is being exerted from one engine, having only the pair of swivel angles, beta pitch and beta yaw, the actual results of the control equation. Although the thrust is so called "single barrel", everything concerning each individual engine (chamber pressure, throat and exit area, etc.) is carried separately and independently with the resulting thrusts being summed to form total thrust. This three-degree-of-freedom trajectory is an instantaneous steady-state solution in pitch and yaw, with no roll taken into account.

The observed trajectory data fall into two major categories, onboard values (telemetry) and ground-fixed measurements. The ground-fixed measurements include position, velocity, and acceleration from UDOP, AZUSA, CZR, theodolite, and radar tracking systems, also single station measurements such as azimuth, elevation, range, range rate, doppler frequency, etc. The telemetry values include integrated guidance accelerations and velocities, radar altimeter, etc. With continuous telemetry coverage and a 0.1 second sampling rate in addition to the ground-fixed measurements, the long burning time leads to large data processing and handling difficulties. The problem of the gigantic number of possible observations is handled by utilizing variable record length for the observation tape. This is a time-sequenced binary magnetic tape with each observation being identified as to type and origin. For each observation an external estimate of the noise in the data is used as a weight in the solution and these values accompany the observations on the input tape. Observations which occur in groups, such as position (X, Y, Z), etc., are handled as one type on the observation tape, but are used as individual observations.

All the observed data from the various tracking systems have their own point of reference on the vehicle; for example the UDOP positions are referenced to the UDOP antenna. The solution of the differential equations from the numerical integration gives a time history of the center of gravity (CG) position of the vehicle. In order that the residual be presented in the original observation system and to prevent a systematic error due to the difference in position, a tracking point translation is incorporated to translate the position at any time from the CG to whatever observation tracking point is necessary before the residual or partial derivatives are formed. Also the tracking point for each system is initialized at time equal zero, so that the initial position for all systems read zero.

Either observed or computed meteorological data may be used. If observed meteorological data are used, they are taken from a fixed-record-length binary magnetic tape which is sequenced by ascending altitudes. The information on the tape is obtained at, or close to, the time of firing and contains atmospheric pressure, density, wind velocity, wind direction, velocity of sound, and of course altitude.

Since there exists no closed-form solution to the equations of motion, the partial derivatives for the adjustment procedure are obtained by a method of numerical perturbation. The parameters chosen for adjustment are thrust, mass, rate of change of mass, beta pitch, and beta yaw, the latter two being the two swivel angles. The correction for each parameter is in the form of a quadratic polynomial in time, each coefficient of the polynomial being a different parameter. The PR+1 trajectories required by this method are integrated simultaneously, thus allowing summation of the adjustment matrices as the partials and residuals are generated. The size of matrices is controlled by symbolic programming. The correction procedure is an iterative type and the partials are recomputed for each iteration.

In addition to correcting the parameters already noted, the values of the coefficient of drag are solved for during each iteration and are used in the following iteration. The observed values of longitudinal acceleration are used in the computation and therefore they are required to be accurate, as well as noise and bias free, in order for the coefficient of drag correction to be accomplished.

Due to the correlation between thrust, mass, and mass dot, some scheme or constraint has to be used before realistic values of these parameters can be solved for in the correction procedure. At the present time, constraint equations are used for this purpose. Assuming that the initial value of these parameters are known with an accuracy of one percent, the constraint equations then become a function of the number of observations and the accuracy of the value of the parameter itself. The constraints will be explained in detail later. More work needs to be done in this area.

SECTION II. EQUATIONS ✗* FOR A STANDARD THREE-DEGREE-OF-FREEDOM TRAJECTORY

A. PRECOMPUTED VALUES

$$\psi_o = \tan^{-1}\left[\frac{b^2}{a^2} \tan \Phi_o\right]$$

$$\beta_o = \Phi_o - \psi_o$$

$$R_o = a\sqrt{\frac{\mu - 1}{\mu \cos (2\psi_o) - 1}}$$

$$\mu = \frac{a^2 - b^2}{a^2 + b^2}$$

$$\Omega_1^{(1)} = -\cos \Phi_o \sin K$$

$$\Omega_2^{(1)} = \sin \Phi_o$$

$$\Omega_3^{(1)} = \cos \Phi_o \cos K$$

*✗ Arrived at from a computer program in conjunction with references [1] and [6]

$$\Omega_1 = -\omega \cos \Phi_o \sin K$$

$$\Omega_2 = \omega \sin \Phi_o$$

$$\Omega_3 = \omega \cos \Phi_o \cos K$$

$$X_o = R_o \sin \beta_o \sin K$$

$$Y_o = R_o \cos \beta_o$$

$$Z_o = -R_o \sin \beta_o \cos K$$

$$\dot{X}_o = -\omega \cos K (R_o \cos \psi_o + H_o \cos \Phi_o)$$

$$\dot{Y}_o = 0$$

$$\dot{Z}_o = -\omega \sin K (R_o \cos \psi_o + H_o \cos \Phi_o)$$

$$\dot{\xi}_o = \dot{X}_o \cos \epsilon - \dot{Z}_o \sin \delta_1 \sin \epsilon$$

$$\dot{\eta}_o = -\dot{X}_o \sin \epsilon - \dot{Z}_o \sin \delta_1 \cos \epsilon$$

$$\dot{\zeta}_o = \dot{Z}_o \cos \delta_2$$

B. COMPUTATION OF THE DIFFERENTIAL EQUATIONS OF MOTION

The gravitational and the measured components of the acceleration are integrated separately. The first step is to sum the components of displacement and velocity. The resultant sums give the magnitudes of the displacement and velocity components of the vehicle in the space-fixed (X_p, Y_p, Z_p) coordinate system with origin at the initial position of the launch pad.

$$\dot{X} = \dot{X}_g + \dot{X}_m + \dot{X}_o$$

$$\dot{Y} = \dot{Y}_g + \dot{Y}_m + \dot{Y}_o$$

$$\dot{Z} = \dot{Z}_g + \dot{Z}_m + \dot{Z}_o$$

$$X = \dot{X}_o T + X_g + X_m$$

$$Y = \dot{Y}_o T + Y_g + Y_m + H_o$$

$$Z = \dot{Z}_o T + Z_g + Z_m$$

To translate from a space-direction-fixed, right-handed system with the origin at the initial position of the launch site to a space-direction-fixed, right-handed system with the origin at the center of the earth.

The following equations are used:

$$X'' = X + X_o$$

$$Y'' = Y + Y_o$$

$$Z'' = Z + Z_o$$

$$R = \sqrt{(X'')^2 + (Y'')^2 + (Z'')^2}$$

The sine of the geocentric latitude (ψ) at this instantaneous time is computed by the expression for the scalar product between the unit vector of the angular velocity of the earth and the position vector of the vehicle.

$$\sin \psi = \frac{1}{R} \left(\Omega_1^{(1)} X'' + \Omega_2^{(1)} Y'' + \Omega_3^{(1)} Z''\right)$$

$$\cos \psi = \sqrt{1 - \sin^2 \psi}$$

$$\psi = \tan^{-1} \left[\frac{\sin \psi}{\cos \psi} \right]$$

$$\Phi = \tan^{-1} \left[\frac{a^2}{b^2} \frac{\sin \psi}{\cos \psi} \right]$$

The component of gravitation in the direction of the radius vector (R).

$$G_\rho = C10 \left(\frac{a}{R}\right)^2 + C11 \left(\frac{a}{R}\right)^4 \left[C21 \sin^2 \psi + C20\right] +$$

$$C12 \left(\frac{a}{R}\right)^6 \left[C32 \sin^4 \psi + C31 \sin^2 \psi + C30\right]$$

The component of gravitation in the direction normal to the radius vector (R), in the plane containing the latitude, ψ.

$$G_\psi = B10\ \left(\frac{a}{R}\right)^4 \sin\psi \cos\psi + \left(\frac{a}{R}\right)^6 \left[B21 \sin^2\psi + B20 \right] \sin\psi \cos\psi$$

The gravitational components of acceleration

$$\ddot{X}_g = G_\rho \frac{X''}{R} + \frac{G_\psi}{R^2 \cos\psi} \left[\Omega_1^{(1)} \left[(Y'')^2 + (Z'')^2 \right] - X'' \left[Y''\ \Omega_2^{(1)} + Z''\ \Omega_3^{(1)} \right] \right]$$

$$\ddot{Y}_g = G_\rho \frac{Y''}{R} + \frac{G_\psi}{R^2 \cos\psi} \left[\Omega_2^{(1)} \left[(X'')^2 + (Z'')^2 \right] - Y'' \left[X''\ \Omega_1^{(1)} + Z''\ \Omega_3^{(1)} \right] \right]$$

$$\ddot{Z}_g = G_\rho \frac{Z''}{R} + \frac{G_\psi}{R^2 \cos\psi} \left[\Omega_3^{(1)} \left[(X'')^2 + (Y'')^2 \right] - Z'' \left[X''\ \Omega_1^{(1)} + Y''\ \Omega_2^{(1)} \right] \right]$$

The distance along the radius vector from the Earth's center to the Earth's surface

$$R_I = a \sqrt{\frac{\mu - 1}{\mu \cos(2\psi) - 1}}$$

The distance along the same radius vector from the Earth's surface to the vehicle is approximately equal to the altitude (H)

$$H = R - R_I$$

The components of the relative velocity

$$V_1 = \dot{X} - (\Omega_2 Z'' - \Omega_3 Y'') - W_1$$

$$V_2 = \dot{Y} - (\Omega_3 X'' - \Omega_1 Z'') - W_2$$

$$V_3 = \dot{Z} - (\Omega_1 Y'' - \Omega_2 X'') - W_3$$

Relative velocity

$$V_R = \sqrt{V_1^2 + V_2^2 + V_3^2}$$

Mach Number

$$Mach = V_R / V_S$$

Direction cosines of the relative velocity

$$V_1^{(1)} = V_1 / V_R$$

$$V_2^{(1)} = V_2 / V_R$$

$$V_3^{(1)} = V_3 / V_R$$

Dynamic Pressure

$$Q = \frac{\rho}{2} \; V_R^2$$

The magnitudes of the measured components of displacement along the ξ, η, ζ axis.

$$\xi_m = X_m \cos \epsilon + (Y_m \cos \delta_1 - Z_m \sin \delta_1) \sin \epsilon$$

$$\eta_m = -X_m \sin \epsilon + (Y_m \cos \delta_1 - Z_m \sin \delta_1) \cos \epsilon$$

$$\zeta_m = Y_m \sin \delta_2 + Z_m \cos \delta_2$$

The measured components of the velocity in the ξ, η, ζ system.

$$\dot{\xi}_m = \dot{X}_m \cos \epsilon + (\dot{Y}_m \cos \delta_1 - \dot{Z}_m \sin \delta_1) \sin \epsilon$$

$$\dot{\eta}_m = - \dot{X}_m \sin \epsilon + (\dot{Y}_m \cos \delta_1 - \dot{Z}_m \sin \delta_1) \cos \epsilon$$

$$\dot{\zeta}_m = \dot{Y}_m \sin \delta_2 + \dot{Z}_m \cos \delta_2$$

The magnitudes of the gravitational components of displacement along the ξ, η, ζ axis.

$$\xi_g = X_g \cos \epsilon + (Y_g \cos \delta_1 - Z_g \sin \delta_1) \sin \epsilon$$

$$\eta_g = -X_g \sin \epsilon + (Y_g \cos \delta_1 - Z_g \sin \delta_1) \cos \epsilon$$

$$\zeta_g = Y_g \sin \delta_2 + Z_g \cos \delta_2$$

The gravitational components of the velocity in the ξ, η, ζ system.

$$\dot{\xi}_g = \dot{X}_g \cos \epsilon + (\dot{Y}_g \cos \delta_1 - \dot{Z}_g \sin \delta_1) \sin \epsilon$$

$$\dot{\eta}_g = - \dot{X}_g \sin \epsilon + (\dot{Y}_g \cos \delta_1 - \dot{Z}_g \sin \delta_1) \cos \epsilon$$

$$\dot{\zeta}_g = \dot{Y}_g \sin \delta_2 + \dot{Z}_g \cos \delta_2$$

$$\xi_m = \xi_m + I_1 T + I_2$$

$$\dot{\xi}_m = \dot{\xi}_m + I_1$$

$$\xi_g = \xi_g - I_1 T - I_2$$

$$\dot{\xi}_g = \dot{\xi}_g - I_1$$

$$\eta_m = \eta_m + J_1 T + J_2$$

$$\dot{\eta}_m = \dot{\eta}_m + J_1$$

$$\eta_g = \eta_g - J_1 T - J_2$$

$$\dot{\eta}_g = \dot{\eta}_g - J_1$$

The magnitudes of the total components of displacement and velocity components along the ξ, η, ζ axis.

$$\xi = \xi_m + \xi_g + \dot{\xi}_o T$$

$$\eta = \eta_m + \eta_g + \dot{\eta}_o T$$

$$\zeta = \zeta_m + \zeta_g + \dot{\zeta}_o T$$

$$\dot{\xi} = \dot{\xi}_m + \dot{\xi}_g + \dot{\xi}_o$$

$$\dot{\eta} = \dot{\eta}_m + \dot{\eta}_g + \dot{\eta}_o$$

$$\dot{\zeta} = \dot{\zeta}_m + \dot{\zeta}_g + \dot{\zeta}_o$$

Magnitude of the components of aerodynamic force along the vehicle's axis.

$$\text{DRAG} = A = C_{DO} \left(\frac{\pi D^2}{4}\right) \frac{\rho}{2} V_R^2 - 75 H + 4500$$

For H $<$ 40 meters

$$\text{DRAG} = A = C_{DO}\left(\frac{\pi D^2}{4}\right)\frac{\rho}{2}V_R^2 + 1500 \Big/ \left[1 + 1000\,(\text{Mach})^4\right]$$

For H $>$ 40 meters

Magnitude of the components of thrust where the i refers to the engine number.

$$F(0) = \sum\left[\left(C_{Fi}\,A_{Ti}\,P_{Ci} - A_{ei}\,P\right)\cos 6^\circ + F_{ei}(0)\cos 6^\circ\right];\quad i=1,2,3,4$$

$$F(I) = \sum\left[\left(C_{Fi}\,A_{Ti}\,P_{Ci} - A_{ei}\,P\right)\cos 3^0 + F_{ei}(I)\cos 60^\circ\right];\quad i=5,6,7,8$$

The restoring moment coefficient due to thrust.

$$(K_2)_p = F(0)\left[\frac{CG - CC}{MOI}\right]$$

$$(K_2)_y = (K_2)_p$$

The restoring moment coefficient due to angle-of-attack.

$$K_1 = (CG - CP)\,C_n'\left(\frac{\pi D^2}{4}\right)\left(\frac{\rho}{2}V_R^2\right)\Big/\ MOI$$

Newton's approximation formula is used in computing the pitch angle of attack.

$$(\alpha_\xi)_{n+1} = (\alpha_\xi)_n - f\,(\alpha_\xi)_n \Big/ f'\,(\alpha_\xi)_n$$

$$f\,(\alpha_\xi)_n = \alpha_\xi + (\chi - \bar{\chi})\,\frac{A_o}{\dfrac{K_1}{(K_2)_p} + B_o} + \frac{C_o\,(\eta - \bar{\eta}) + C_1\,(\dot{\eta} - \dot{\bar{\eta}})}{\dfrac{K_1}{(K_2)_p} + B_o}$$

$$f'\,(\alpha_\xi)_n = 1 + 2\left\{\frac{A_o}{\dfrac{K_1}{(K_2)_p} + B_o}\left[\frac{\dfrac{(-\sin\alpha_\xi)\,\bar{L}}{\bar{K}} + (\bar{K} - V_1^{(1)})}{(\bar{L})^2 + (\bar{K})^2}\right]\text{Cos}\,\alpha_\xi\right\}$$

$$\chi = 2 \tan^{-1} \left[\frac{\bar{K} - V_1^{(1)}}{\bar{L}} \right]$$

$$\bar{K} = \sqrt{(-\sin \alpha_\xi)^2 - \left[(V_1^{(1)})^2 + (V_2^{(1)})^2 \right]}$$

$$\bar{L} = V_2^{(1)} - \sin \alpha_\xi$$

The pitch deflection angle

$$\beta_\xi = - \frac{K_1}{(K_2)_p} \alpha_\xi$$

The angle in the roof plane between the projection of the velocity vector and the X_p, Y_p plane.

$$\nu = \tan^{-1} \left[\frac{V_3^{(1)}}{V_1^{(1)} \cos \chi + V_2^{(1)} \sin \chi} \right]$$

Yaw angle of attack

$$\alpha_\zeta = \frac{A_o \ \nu + E_o \ \zeta_m + E_1 \ \dot{\zeta}_m}{- \frac{K_1}{(K_2)_y} - B_o - A_o}$$

The yaw deflection angle

$$\beta_\zeta = - \frac{K_1}{(K_2)_y} \alpha_\zeta$$

Roof angle

$$\tau = \nu + \alpha_\zeta$$

Direction cosines of the vector in the direction of the vehicle axis in the X'', Y'', Z'' system.

$$C_1^{(1)} = \cos \chi \cos \tau$$

$$C_2^{(1)} = \sin \chi \cos \tau$$

$$C_3^{(1)} = \sin \tau$$

The direction cosines of the outboard chamber thrust vector in the X", Y", Z" system.

$$F_1^{(1)} = \cos \beta_\xi \cos \beta_\zeta \cos \tau \cos \chi - \sin \beta_\zeta \sin \tau \cos \chi - \sin \beta_\xi \cos \beta_\zeta \sin \chi$$

$$F_2^{(1)} = \cos \beta_\xi \cos \beta_\zeta \cos \tau \sin \chi - \sin \beta_\zeta \sin \tau \sin \chi + \sin \beta_\xi \cos \beta_\zeta \cos \chi$$

$$F_3^{(1)} = \cos \beta_\xi \cos \beta_\zeta \sin \tau + \sin \beta_\zeta \cos \tau$$

The angle of attack (α the angle between the vehicle axis and the velocity vector).

$$\cos \alpha = V_1^{(1)} \cdot C_1^{(1)} + V_2^{(1)} \cdot C_2^{(1)} + V_3^{(1)} \cdot C_3^{(1)}$$

$$\sin \alpha = \sqrt{1 - (\cos \alpha)^2}$$

$$\alpha = \tan^{-1} \left[\frac{\sin \alpha}{\cos \alpha}\right]$$

The direction cosines of the normal force vector in the X", Y", Z" system.

$$N_1^{(1)} = \left(C_1^{(1)} \cos \alpha - V_1^{(1)}\right) \Big/ \sin \alpha$$

$$N_2^{(1)} = \left(C_2^{(1)} \cos \alpha - V_2^{(1)}\right) \Big/ \sin \alpha$$

$$N_3^{(1)} = \left(C_3^{(1)} \cos \alpha - V_3^{(1)}\right) \Big/ \sin \alpha$$

The slope of the normal force

$$N' = C_n' \left(\frac{\pi D^2}{4}\right) \frac{\rho}{2} V_R^2$$

The measured components of the acceleration

$$\ddot{X}_m = F_1^{(1)} \left(\frac{F(O)}{M}\right) + C_1^{(1)} \left(\frac{F(I)}{M} - \frac{A}{M}\right) + N_1^{(1)} \left(\frac{N'}{M} \alpha\right)$$

$$\ddot{Y}_m = F_2^{(1)} \left(\frac{F(O)}{M}\right) + C_2^{(1)} \left(\frac{F(I)}{M} - \frac{A}{M}\right) + N_2^{(1)} \left(\frac{N'}{M} \alpha\right)$$

$$\ddot{Z}_m = F_3^{(1)} \left(\frac{F(O)}{M}\right) + C_3^{(1)} \left(\frac{F(I)}{M} - \frac{A}{M}\right) + N_3^{(1)} \left(\frac{N'}{M} \alpha\right)$$

The longitudinal load factor

$$A_L = \frac{F(O)}{M} \cos \beta_\xi \cos \beta_\zeta + \left(\frac{F(I)}{M} - \frac{A}{M}\right)$$

Total acceleration

$$\ddot{X} = \ddot{X}_g + \ddot{X}_m$$

$$\ddot{Y} = \ddot{Y}_g + \ddot{Y}_m$$

$$\ddot{Z} = \ddot{Z}_g + \ddot{Z}_m$$

SECTION III. CORRECTION PROCEDURE

The parameters corrected are thrust, mass, mass dot, beta pitch, and beta yaw. With the assumption that the correction should not be limited to a constant term, a quadratic in time (t) is set up for each parameter and any or all terms of these expressions may or may not be used in the correction. Now the parameters take on the following form:

$$F = \text{Thrust} = F(O) + F(I) + F_o + F_1T + F_2T^2$$

$$M = \text{Mass} = M + M_o + M_1T + M_2T^2$$

$$\dot{M} = \text{Mass Dot} = \dot{M} + \dot{M}_o + \dot{M}_1T + \dot{M}_2T^2$$

$$\beta_p = \text{Beta Pitch} = \beta_\xi + B_{po} + B_{p1}T + B_{p2}T^2$$

$$\beta_y = \text{Beta Yaw} = \beta_\zeta + B_{yo} + B_{y1}T + B_{y2}T^2$$

This gives a flexibility of 24 possible partial derivatives, a constant, linear, and/or quadratic term in each parameter. The partial derivatives are obtained by the method of numerical perturbations. In addition to the standard, a perturbed trajectory must be computed for every partial derivative needed in the correction procedure. Therefore, PR+1 trajectories are computed simultaneously, thus allowing the total equation to be formed at one time.

These parameters and time, in addition to the assumptions made in the basic computation, uniquely determine the trajectory and also any observation of the trajectory. Thus any observation, α_i, is a function of the parameters, $P(F, M, \dot{M}, \beta_p, \beta_y)$, and the time of observation (t).

$\alpha_i = f_i\ (P_J, t)$ Where i refers to the type of observation
J refers to the J'th parameter

By the Taylor expansion, then, neglecting second and higher order terms,

$$\alpha_i = f_i\ (P_J^o,\ t) + \left[\sum_J \frac{\partial f_i}{\partial P_J} \cdot \triangle P_J \right]_{P_J^o}$$

where P_J^o is a set of approximate initial values of the parameters.

For a series of N observations (α_i), a set of N simultaneous equations exists for the solution of the $\triangle P_J$'s required to adjust the initial parameters to match the observations. The total differential of an observation in terms of the chosen parameters

$$\frac{\partial\alpha_i}{\partial F_o} \triangle F_o + \frac{\partial\alpha_i}{\partial F_1} \triangle F_1 + \ldots + \frac{\partial\alpha_i}{\partial M_o} \triangle M_o + \ldots + \frac{\partial\alpha_i}{\partial \dot{M}_1} \triangle \dot{M}_1 + \ldots . + = \triangle\alpha_i$$

where $\dfrac{\partial\alpha_i}{\partial P_J}$ $\approx$ the value of the observation computed from the perturbed trajectory minus the value computed with an unperturbed trajectory, divided by the perturbation size.

$\triangle\ \alpha_i$ = the actual observed value minus the value computed with an unperturbed trajectory.

Denoting the matrix of partials $\dfrac{\partial\alpha_i}{\partial P_J}$ by $\overline{B}$, the matrix of $\triangle$'s $(\triangle P_J)$ by $\overline{\triangle}$ and the matrix of residuals by $\overline{C}$, in matrix form we have;

$$\overline{B}\ \overline{\triangle} = \overline{C}$$

If there are more observations (α_i) than unknowns ($\triangle P_J$) the system is overdetermined, and the equations are solved by the method of least-squares.

$$\bar{\triangle} = \left[\bar{B}^T \bar{B} \right]^{-1} \bar{B}^T \bar{C}$$

The least-squares differential correction is a standard general matrix solution encompassing multivariate observations. With the variances of all the observations not the same, the solution now becomes

$$\bar{\triangle} = \left[\bar{B}^T \bar{w} \bar{B} \right]^{-1} \bar{B}^T \bar{W} \bar{C}$$

where $\bar{w}$ is a weight matrix such that

$$\bar{w} = 1/\sigma\alpha_i^2$$

$\sigma\alpha_i$ is the standard deviation of the data point, α_i. Correlation between observations is assumed to be zero, therefore the off-diagonal terms of the weight matrix are zero.

The computed corrections, $\triangle P_J$, are applied to the approximate parameters, P_J, to yield improved values, P_J^o, which are then used as approximate parameters in the correction process, and new $\triangle P_J$'s are computed. This iteration process is repeated until the corrections converge ($\triangle P_J \rightarrow 0$).

SECTION IV. CONSTRAINTS

The present approach to the problem of correlation between the chosen parameters is admittedly not the best. Due to the original formulation of the problem the relationship between the parameters was not taken into consideration in the solution. Therefore some type of "damping" of the least-squares adjustment matrix was necessary in order to get physically realistic values of the parameters from the correction. It is known at the present time that either thrust or mass dot can be eliminated from the solution since one is a function of the other. It is hoped at a later time that this can be incorporated properly. The damping method is similar to that incorporated by Levenberg[3] and Foster[4].

The following matrix is added to the least-squares adjustment matrix.

$$\begin{pmatrix} \left(\frac{N}{\sigma P_1}\right)^2 & 0 & \cdots & 0 \\ 0 & \left(\frac{N}{\sigma P_2}\right)^2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \left(\frac{N}{\sigma P_J}\right) \end{pmatrix}$$

Where N -Number of observation equations

σP_J -Accuracy estimate of the value of the parameter.

SECTION V. COEFFICIENT OF DRAG CORRECTION

When the first preliminary test corrections were performed, it was found that the first approximation to the trajectory was very poor and that the corrected trajectory residuals contained characteristics which could be correlated with the coefficient of drag. It was decided at a later time that a corrected coefficient of drag should be obtained through the usage of the observed longitudinal acceleration measurement from telemetry. The coefficient of drag is utilized in table form as a function of mach number. A three point Lagrange interpolation is used to solve for the time that a specific mach number occurred. A similar interpolation is then performed to find the observed longitudinal acceleration for this particular time. Utilizing all quantities for this time, the value of coefficient of drag is obtained and placed in a table. At the end of the iteration this table of C_{DO} values along with the corrected parameters are used to generate the trajectories for the next iteration. The C_{DO} values are obtained by the following formulation.

$$A_L = \frac{F(O)}{M} \cos \beta_\xi \cos \beta_\zeta + \frac{F(I)}{M} - \frac{A}{M}$$

Where $A = C_{DO} \left(\frac{\pi D^2}{4}\right) \frac{\rho V_R^2}{2} + C$

$C = -75\ H + 4500$ For $H < 40$ meters

$C = 1500/[1 + 1000\ (\text{Mach})^4]$ For $H > 40$ meters

$$\text{Hence } C_{DO} = \frac{(F(O)\cos\beta_\xi \cos\beta_\zeta + F(I) - M\,A_L - C)^8}{\pi\, D^2\, \rho V_R^2}$$

SECTION VI. TRACKING POINT INITIALIZATION

At time equal zero it is necessary to determine the location of the tracking point for each system with reference to the center of gravity of the vehicle. This initial difference is substracted from the CG-referenced coordinates when they are translated to the observation system. This eliminates the bias that results from the fact that the tracking point does not coincide with the center of gravity. The initial position of the point of track for the i th system with reference to the CG:

$$\Delta X_i = R_i \cos \theta_i$$

$$\Delta Y_i = (h_{tp})_i - h_{CG}$$

$$\Delta Z_i = R_i \sin \theta_i$$

where R_i Radial distance from center line of vehicle to tracking point.

θ_i Angle ref. to fin I measured positively in the same direction as positive Φ_r.

$(h_{TP})i$ Height of tracking point from station 0.

h_{CG} Height of center of gravity from station 0.

Translation from CG to the Tracking Point

The following is the space-fixed transformation used to translate, at any time (t) from the center of gravity to the i th tracking point.

$$\Delta X_i = (h_{TPi} - h_{CG}) \cos \chi \cos \tau + R_i \cos \theta_i \sin \chi \cos \tau$$

$$\Delta Y_i = (h_{TPi} - h_{CG}) \sin \chi \cos \tau - R_i \cos \theta_i \cos \chi \operatorname{Cos} \tau$$

$$\Delta Z_i = (h_{TPi} - h_{CG}) \sin \tau + R_i \sin \theta_i \cos \tau$$

SECTION VII. ERROR ANALYSIS

With the output from the correction procedure being purposely limited to the printing of the computed corrections (ΔP_J's) for each iteration, an analysis of the correction should be performed. A binary output tape containing the weighted observation equations and inverted matrix is written at the end of the first and last iterations. This gives a "before" and "after" look. The analyses performed are the obtaining of the correlation coefficient between the parameters, standard deviation of the parameters, and the standard deviation of the data by type. An automatic plot tape is generated containing the residuals by data type versus time. This tape is for use with the automatic tape plotter.

There are $\frac{P}{2}$ (P-1) correlation coefficients between the P parameters. Denoting the covariance or inverse least-squares matrix by N^{-1}, the i th diagonal element of the covariance matrix, a_{ii}, is the variance of the i th parameter. The correlation coefficient between P_i and P_J is defined as

$$C_{ij} = \frac{a_{ij}}{\sqrt{a_{ii}}\ \sqrt{a_{jj}}} \qquad i = 1, 2, \ldots, P$$

$$C_{ij} = C_{ji} \qquad j = 1, 2, \ldots, P$$

Standard deviation of the parameters

$$\sigma = \left(\sqrt{\frac{\sum_{1}^{N} (WE)^2}{(N - P)}}\right) \sqrt{a_{ij}} \qquad , i = j$$

Where WE is the weighted residual.
N is the number of observation equations

Standard deviation of the total reduction

$$\sigma = \sqrt{\frac{\sum_{1}^{N} (WE)^2}{(N-P)}}$$

Standard deviation for each data type

$$\sigma_T = \sqrt{\left(\sum_{1}^{N_T} (WE)_T^2 \right) \Big/ (N_T - P)}$$

Where t refers to the data type

SECTION VIII. RESULTS

Here are the results of a differential correction using data from SA-1. Only the constant term of the correction polynominal in each of the parameters is solved for in this correction. The observation data used are UDOP position coordinates. The correction covers the first 108.0 seconds of flight time and did not correct the coefficient of drag curve. The UDOP coordinates are at 0.1 second intervals. Using positions from only one tracking system, the weights were assumed as 1.0. This correction converged at the end of three iterations. FIGURES 1-6 show plots of the "before" and "after" residuals.

Constraints Equations

$$7.4965\ (\Delta M) + 0(\Delta\dot{M}) + 0(\Delta F) + 0(\Delta\beta_y) + 0(\Delta\beta_p) = 0$$

$$0(\Delta M) + 1322.7272\ (\Delta\dot{M}) + 0(\Delta F) - 0(\Delta\beta_y) + 0(\Delta\beta_p) = 0$$

$$0(\Delta M) + 0(\Delta M) + 5.3350\ (\Delta F) + 0(\Delta\beta_y) + 0(\Delta\beta_p) = 0$$

Iteration No.	ΔM	$\Delta\dot{M}$	ΔF	$\Delta\beta_y$(radians)	$\Delta\beta_p$(radians)
1	-121.41	-0.00715	-148.79	-0.00229398	+0.00452002
2	- 14.63	-0.00339	- 34.46	-0.00000817	+0.00003675
3	+ 0.15	-0.00326	+ 0.08	+0.00000153	+0.00000050
Total	-135.89	-0.01380	-183.17	-0.00230062	+0.00455727

Note: To obtain the total correction in thrust, the total ΔF should be multiplied by the number of engines.

SECTION IX. ANALYSIS OF THE CORRECTION

Cross correlation factors from the inverted matrix.

PARAMETERS			ITERATION 1	ITERATION 3
Mass	-	Mass Dot	+0.187944	+0.190493
Mass	-	Thrust	+0.997032	+0.997014
Mass	-	Beta Yaw	+0.000000	-0.036459
Mass	-	Beta Pitch	-0.697678	-0.679976
Mass Dot		Thrust	+0.142444	+0.144746
Mass Dot		Beta Yaw	+0.000000	-0.006945
Mass Dot		Beta Pitch	-0.044545	-0.033068
Thrust		Beta Yaw	+0.000000	-0.036350
Thrust		Beta Pitch	-0.698830	-0.681355
Beta Yaw		Beta Pitch	+0.000000	+0.024791

Standard deviation of the total reduction.

Iteration No.	Sigma
1	61.034901
3	15.543454

Standard deviation of the Parameters.

Parameter	σ (Iteration 1)	σ (Iteration 3)
Mass	0.485789	0.122574
Mass Dot	0.046121	0.011745
Thrust	0.390549	0.098886
Beta Yaw	0.066828	0.017190
Beta Pitch	0.096477	0.024016

Standard deviation of the Data by Type.

Type	σ(Iteration 1)	σ(Iteration 3)
UDOP - X	45.464776	19.509836
UDOP - Y	88.071841	17.737941
UDOP - Z	37.244831	5.639811

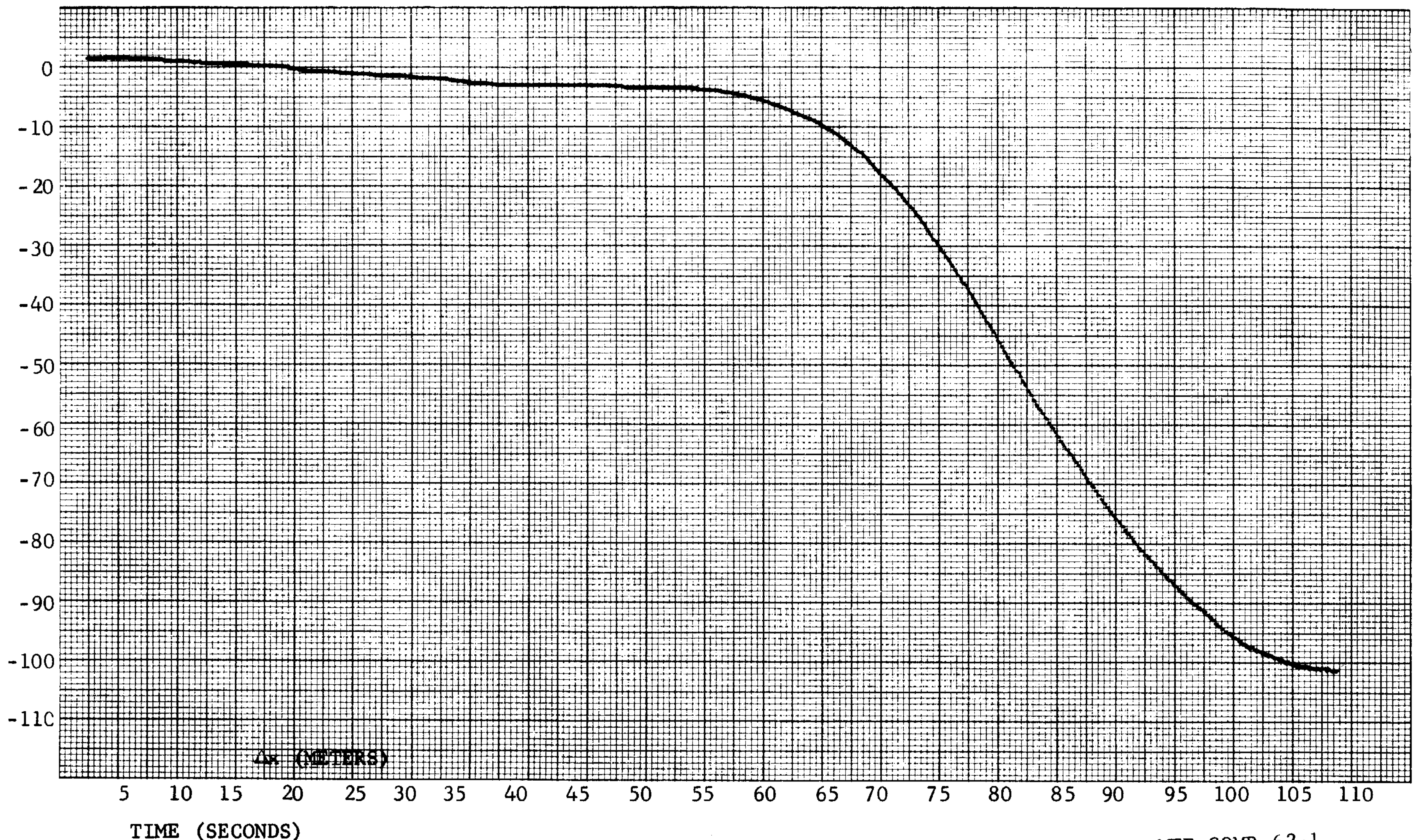

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FIGURE 1. RESIDUAL IN X BEFORE CORRECTION.

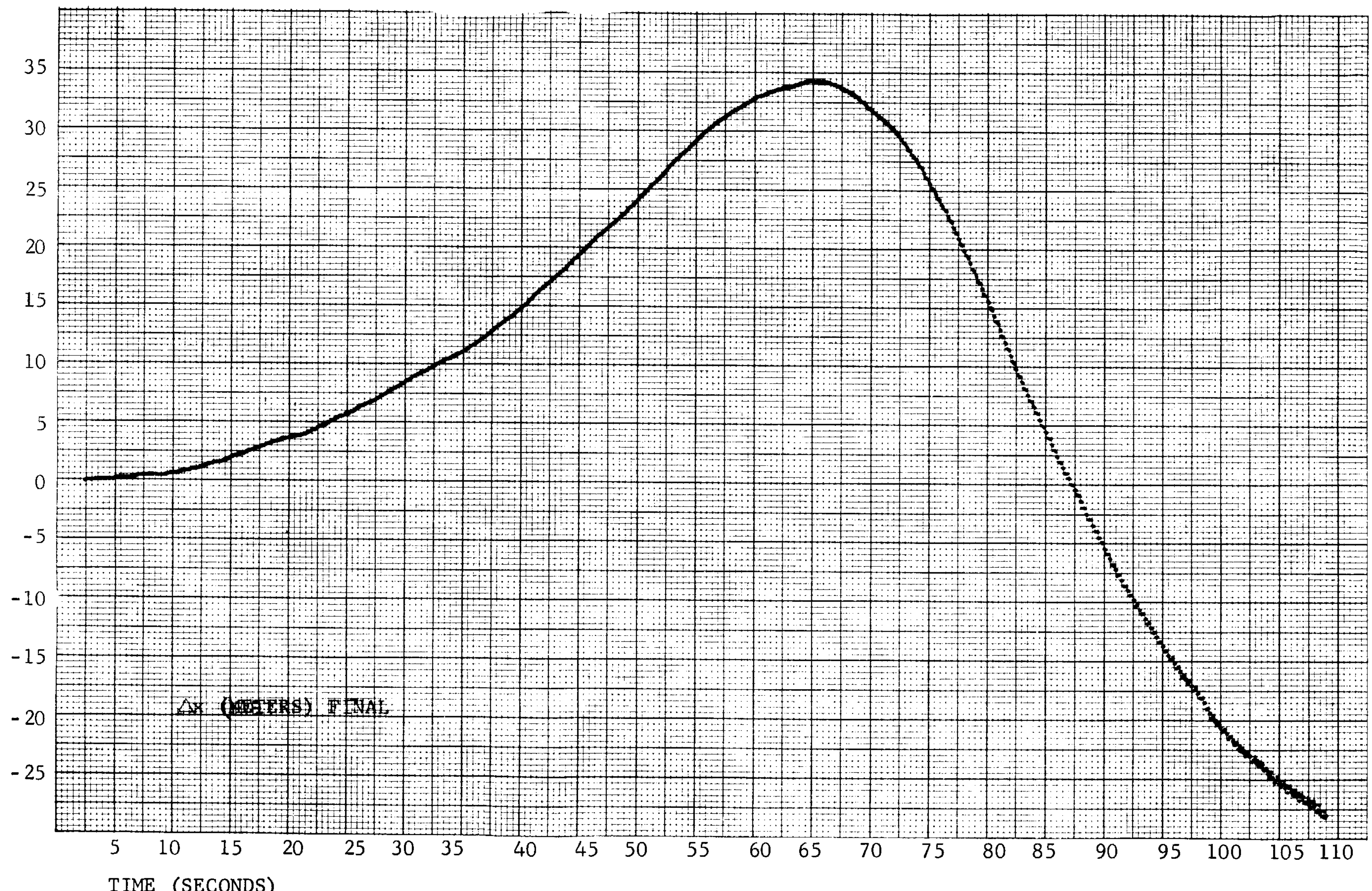

FIGURE 2. RESIDUAL IN X AFTER CORRECTION.

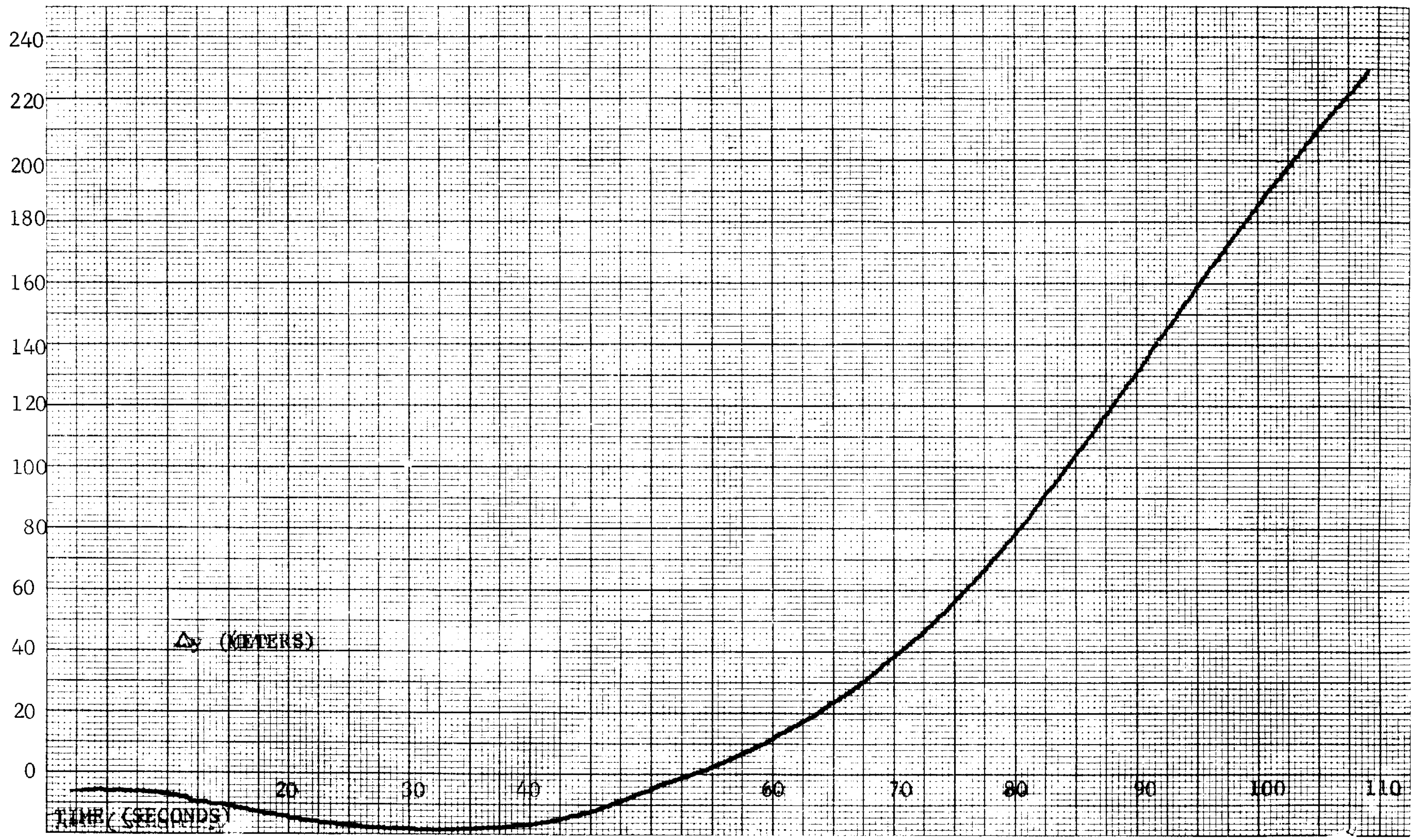

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FIGURE 3. RESIDUAL IN V BEFORE CORRECTION.

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FIGURE 4. RESIDUAL IN Y AFTER CORRECTION.

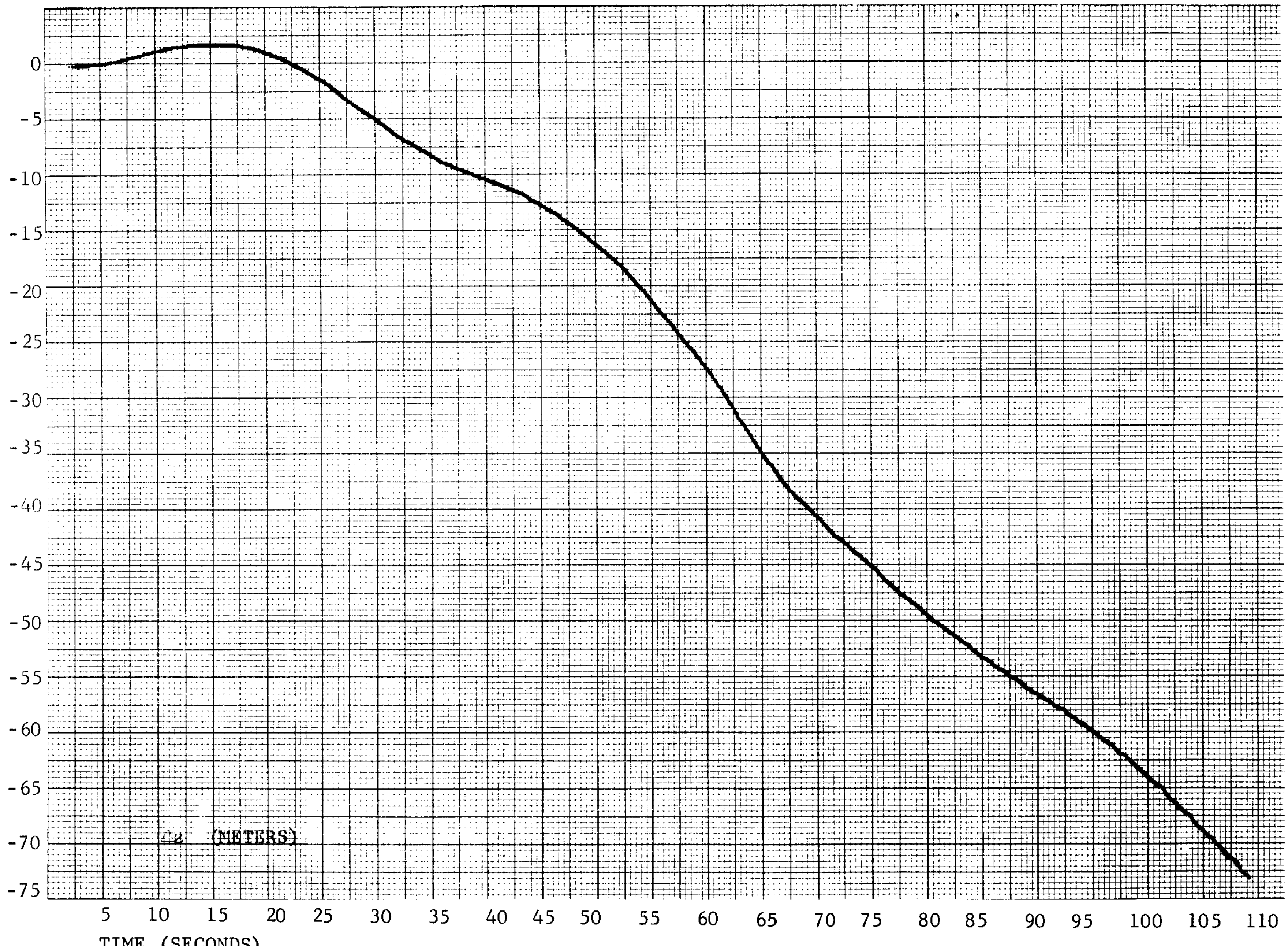

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FIGURE 5. RESIDUAL IN Z BEFORE CORRECTION.

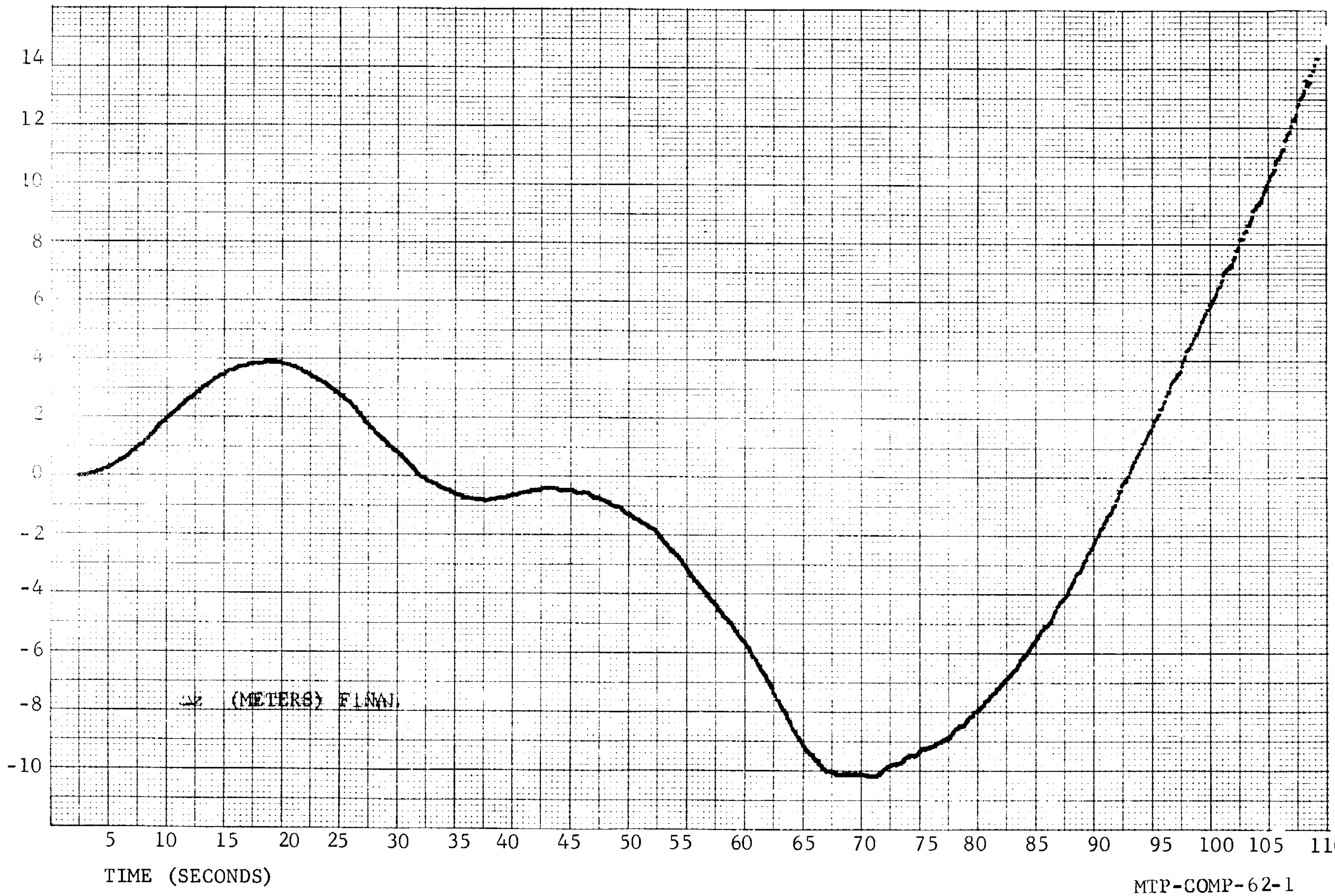

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FIGURE 6. RESIDUAL IN Z AFTER CORRECTION.

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APPENDIX

WINDS*

In general, the direction of wind is parallel to the surface of the earth.

South Wind: The direction of South Wind, $\overline{S}^{(1)}$, is defined as parallel to the surface of the earth and coming from the south as viewed from the instantaneous position of the vehicle.

West Wind: The direction of West Wind, $\overline{\omega}^{(1)}$, is defined as parallel to the surface of the earth and coming from the west as viewed from the vehicle. The wind is always perpendicular to the instantaneous meridian plane of the earth through the missile.

Derivation:

Consider the radius vector, $\overline{R}$, from the center of the earth to the center of gravity of the vehicle. The cross product of this vector and the earth's rotational unit vector $\overline{\omega}^{(1)}$, is the vector $\overline{\sigma}$.

$$\overline{\sigma} = \overline{R} \times \overline{\omega}^{(1)}$$

$$\overline{\sigma}^{(1)} = \frac{\overline{R}^{(1)} \times \overline{\omega}^{(1)}}{\sin (90-\psi)} = \frac{\overline{R}^{(1)} \times \overline{\omega}^{(1)}}{\cos \psi}$$

$\overline{\sigma}^{(1)}$ defines the direction of the angular velocity vector of a south wind. The direction of the south wind itself, $\overline{S}^{(1)}$, is the cross product of $\overline{\sigma}^{(1)}$ and $\overline{R}^{(1)}$.

$$\overline{S}^{(1)} = \overline{\sigma}^{(1)} \times \overline{R}^{(1)} \qquad \overline{\sigma}^{(1)} \perp \overline{R}^{(1)}$$

$$\overline{S}^{(1)} = \frac{1}{\cos \psi} \left[(\overline{R}^{(1)} \times \overline{\omega}^{(1)}) \times \overline{R}^{(1)} \right]$$

The cross product $\overline{\omega}^{(1)} \times \overline{R}^{(1)}$ defines the direction of the west wind unit vector $\overline{w}^{(1)}$.

$$\overline{w}^{(1)} = \frac{\overline{\omega}^{(1)} \times \overline{R}^{(1)}}{\cos \psi} = - \frac{(\overline{R}^{(1)} \times \overline{\omega}^{(1)})}{\cos \psi} = - \frac{1}{\cos \psi} (\overline{R}^{(1)} \times \overline{\omega}^{(1)})$$

*Taken from Ref. 6

The wind directions have been derived in cardinal headings. It is now desired to resolve these winds about the firing azimuth, K, measured counterclockwise from west. The unit wind vector may be expressed as:

$$\bar{W}^{(1)} = DC\ (\bar{S}^{(1)} \sin k + \bar{w}^{(1)} \cos k) + EC\ (\bar{S}^{(1)} \cos k - \bar{w}^{(1)} \sin k)$$

where DC and EC specify the wind direction. Now translate the wind vector into space-fixed coordinates.

$$\bar{w}^{(1)} = - \frac{1}{\cos \psi} \left[\bar{R}^{(1)} \times \bar{w}^{(1)} \right] = - \frac{1}{R \cos \psi} \begin{pmatrix} \bar{i} & \bar{j} & \bar{k} \\ X'' & Y'' & Z'' \\ \Omega_1^{(1)} & \Omega_2^{(1)} & \Omega_3^{(1)} \end{pmatrix}$$

Where $\frac{X''}{R}$, $\frac{Y''}{R}$, $\frac{Z''}{R}$ are the components of $\bar{R}^{(1)}$ in the space-fixed system with origin at the center of the earth, and $\Omega_1^{(1)}$, $\Omega_2^{(1)}$, $\Omega_3^{(1)}$ are the components of $\bar{\omega}^{(1)}$.

$$\bar{w}^{(1)} = - \frac{1}{R \cos \psi} \Big[(\Omega_3^{(1)} Y'' - \Omega_2^{(1)} Z'') \bar{i} + (\Omega_1^{(1)} Z'' - \Omega_3^{(1)} X'') J +$$

$$+ (\Omega_2^{(1)} X'' - \Omega_1^{(1)} Y'') \bar{k} \Big]$$

$$\bar{w}_1^{(1)} = - \frac{1}{R \cos \psi} (\Omega_3^{(1)} Y'' - \Omega_2^{(1)} Z'')$$

$$w_2^{(1)} = - \frac{1}{R \cos \psi} (\Omega_1^{(1)} Z'' - \Omega_3^{(1)} X'')$$

$$w_3^{(1)} = - \frac{1}{R \cos \psi} (\Omega_2^{(1)} X'' - \Omega_1^{(1)} Y'')$$

$$\bar{S}^{(1)} = \frac{1}{\cos \psi} \left[(\bar{R}^{(1)} \times \bar{\omega}^{(1)}) \times \bar{R}^{(1)} \right]$$

$$\bar{S}^{(1)} = \frac{1}{R^2 \cos \psi} \begin{pmatrix} \bar{i} & \bar{j} & \bar{k} \\ \bar{w}_1^{(1)} & \bar{w}_2^{(1)} & \bar{w}_3^{(1)} \\ X'' & Y'' & Z'' \end{pmatrix}$$

$$\bar{S}^{(1)} = \frac{1}{R^2 \cos \psi} \Big[(Z'' w_2^{(1)} - Y'' w_3^{(1)}) \bar{i} + (X'' w_3^{(1)} - Z'' w_1^{(1)}) \bar{j} +$$

$$+ (Y'' W_1^{(1)} - X'' W_2^{(1)}) \bar{k} \Big]$$

$$S_1^{(1)} = \frac{1}{R^2 \cos \psi} \left[Z'' w_2^{(1)} - Y'' w_3^{(1)} \right]$$

$$S_2^{(1)} = \frac{1}{R^2 \cos \psi} \left[X'' w_3^{(1)} - Z'' w_1^{(1)} \right]$$

$$S_3^{(1)} = \frac{1}{R^2 \cos \psi} \left[Y'' w_1^{(1)} - X'' w_2^{(1)} \right]$$

The total wind vector, $\bar{W}$, is obtained by multiplying the unit wind vector by the scalar W (h).

$$\bar{W} = W\ (h)\ \bar{W}^{(1)}$$

Where W (h) is the wind magnitude as a function of altitude.

The total wind vector, $\bar{W}$, can now be expressed in space-fixed coordinates.

$$\bar{W} = W\ (h) \left[DC\ (\bar{S}^{(1)} \sin k + \bar{w}^{(1)} \cos k) + EC\ (\bar{S}^{(1)} \cos k - \bar{w}^{(1)} \sin k) \right]$$

Therefore the three components of wind become:

$$W_1 = W\ (h) \left[DC\ (S_1^{(1)} \sin k + w_1^{(1)} \cos k) + EC\ (S_1^{(1)} \cos k - w_1^{(1)} \sin k) \right]$$

$$W_2 = W(h) \left[DC \ (S_2^{(1)} \sin k + w_2^{(1)} \cos k) + EC \ (S_2^{(1)} \cos k - w_2^{(1)} \sin k)\right]$$

$$W_3 = W(h) \left[DC \ (S_3^{(1)} \sin k + w_3^{(1)} \cos k) + EC \ (S_3^{(1)} \cos k - w_3^{(1)} \sin k)\right]$$

If observed meterological data is used, wind magnitude, W (h), and wind direction measured positive eastward and North, α_W, are both available as a function altitude and:

$$DC = \cos (270° - \alpha_W - K)$$

$$EC = \sin (270° - \alpha_W - K)$$

If a reference atmosphere is used, the winds are set to zero.

Before 7 seconds, which corresponds to about 75 meters in altitude, the winds are set to zero.

APPROVAL MTP COMP-62-1

TRAJECTORY DETERMINATION BY A LEAST-SQUARES DIFFERENTIAL CORRECTION OF THREE-DEGREE-OF-FREEDOM ACCELERATIONS

By

Paul O. Hurst

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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